

# Modified TOV in gravity's rainbow: properties of neutron stars and dynamical stability conditions

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In this paper, we consider a spherical symmetric metric to extract the hydrostatic equilibrium equation of stars in  $(3+1)$ -dimensional gravity's rainbow in the presence of cosmological constant. Then, we generalize the hydrostatic equilibrium equation to  $d$ -dimensions and obtain the hydrostatic equilibrium equation for this gravity. Also, we obtain the maximum mass of neutron star using the modern equations of state of neutron star matter derived from the microscopic calculations. It is notable that, in this paper, we consider the effects of rainbow functions on the diagrams related to the mass-central mass density ( $M$ - $\rho_c$ ) relation and also the mass-radius ( $M$ - $R$ ) relation of neutron star. We also study the effects of rainbow functions on the other properties of neutron star such as the Schwarzschild radius, average density, strength of gravity and gravitational redshift. Then, we apply the cosmological constant to this theory to obtain the diagrams of  $M$ - $\rho_c$  (or  $M$ - $R$ ) and other properties of these stars. Next, we investigate the dynamical stability condition for these stars in gravity's rainbow and show that these stars have dynamical stability. We also obtain a relation between mass of neutron stars and Planck mass. In addition, we compare obtained results of this theory with the observational data.

## I. INTRODUCTION

In generalization from Galilean Relativity to Special Relativity, one has to modify the kinematic equations of Galilean Relativity to obtain an invariant velocity scale. The same method could be followed in order to derive a theory, containing an invariant energy scale which is Planck scale. Such modification is known as the doubly special relativity [1]. In other words, we take two upper limits for the particles probing a spacetime into account: the speed of light and the Planck energy. Due to modifications in the kinematic structure of the doubly special relativity, the energy-momentum conservation laws and the Lorentz symmetry group are altered. There are several theories suggesting that standard energy-momentum dispersion relation is modified in limit of the Planck scale which among them one can name; string theory [2], loop quantum gravity [3] and non-commutative geometry [4]. The generalization of the doubly special relativity to incorporate curvature is known as gravity's rainbow [1].

The consideration of doubly special relativity, hence gravity's rainbow, is related to the effects of quantum spacetime. In other words, it is proposed that the classical theory of the gravity is an emerging one from quantum degrees of freedom which in result causes the theory to be an effective one [5–9]. Such an effective theory has an energy dependent metric in order to describe different phenomena [10]. The construction of the metric is determined by a particle probing it [11].

On the other hand, the Einstein theory of gravity has fundamental problem in UV limit which indicates that it requires modification. It is not necessary to modify the action in order to have UV completion theory. Alteration of the metric describing the spacetime in some cases would be sufficient. Such an approach is employed in Horava-Lifshitz theory of the gravity as well. The gravity's rainbow also enjoys this feature. In other words, it is an UV completion theory which in IR limit reduces to classical theory of the Einstein gravity. It is worthwhile to mention that by considering proper values for the energy functions of the gravity's rainbow, this theory would yield a model of the Horava-Lifshitz which indicates that these theories are related. The UV completion theories require that their energy-momentum dispersion relation to be modified. Such modification is also observed in discrete spacetime [12], spacetime foam [13], spin-network in loop quantum gravity (LQG) [3], ghost condensation [14] and non-commutative geometry [4]. In addition, experimental observations also confirmed that such modification must be considered in the UV limit [15].

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Several studies are conducted in context of black holes by consideration of the gravity's rainbow [16–21]. It was shown that the thermodynamic properties of black holes are modified in gravity's rainbow in such a way that it admits the existence of remnant for black holes. In evaporation of the black holes in gravity's rainbow, the temperature of black holes goes to zero while its size is finite. Therefore, the black hole is not fully evaporated. Due to this feature, it is proposed that this model could provide a solution regarding the information paradox [22] and the formation of naked singularity at the last stage of the black hole's evaporation. The existence of remnant is observed for Kerr black holes, Kerr-Newman black holes in de Sitter space, charged AdS black holes, higher dimensional Kerr-AdS black holes and black saturn [23]. In addition, due to this phenomena, there exists an energy limit in which the mini black holes can be produced at LHC [24]. It is worthwhile to mention that the gravity's rainbow also holds the usual uncertainty principle [25, 26]. Recently, there has been an increasing interest in gravity's rainbow [27–31]. The wormhole solutions in the context of gravity's rainbow have been investigated in Ref. [32]. A study regarding the extended informal approach in gravity's rainbow is done in Ref. [33]. In addition, the rainbow metric from quantum has been obtained in Ref. [34]. On the other hand, the effects of gravity's rainbow on the thermodynamics of different models of black holes have been investigated in literature [16–21]. Einstein theory of gravity is an effective theory which confronts specific problems in different regimes such as UV or in describing some phenomena such as increasing accelerating expansion of the Universe which was proven by the observation of high red-shift supernova [35–37] and the measurement of angular fluctuations of cosmic microwave background fluctuations [38–41]. One of the best candidates for modifications of Einstein theory is adding the cosmological constant to Einstein's Lagrangian [42, 43].

The physics governing the structure of stars includes the interaction between gravitational force and internal pressure which results into an equilibrium state. This is known as hydrostatic equilibrium equation (HEE) which plays the essential role in studying the structure of stars. On the other hand, it was proven that in studying the compact objects such as the neutron and strange quark stars, consideration of curvature of space-time and generalization of Newtonian gravity to general relativity is necessary for describing different phenomena and making accurate predictions. The first attempt for obtaining Einsteinian HEE for stars was done by Tolman, Oppenheimer and Volkoff (TOV) [44–46]. The TOV approach toward studying the structure of the stars has been employed by several authors [47–55]. In addition, the generalization to modified theories of gravity such as  $F(R)$ ,  $F(G)$  [56–59] and dilaton [60] gravities are done (for more details see Refs. [61–66]).

There has been an ongoing debate regarding the consideration of the higher dimensionality in studying different phenomena. There are several reasons which are supporting such generalization: First of all, in context of particles interactions, the necessity of such generalization is expressed for having consistent theories. On the other hand, it is essential to consider higher dimensionality in order to have an effective theory of superstring [67–69]. The phenomenology of higher dimensionality has proven to be richer [70–72]. As for the stars, a study regarding higher dimensional mass-radius relation for a star with the uniform density was done in Ref. [73]. The consideration of Kaluza-Klein model for describing the neutron stars was done in Ref. [74]. This study highlighted the importance of higher dimensionality. In addition, the relativistic anisotropic stars were investigated, and the effects of higher dimensions were pointed out [75]. The TOV equation of Einstein- $\Lambda$  gravity in higher dimensions was studied in the Refs. [76, 77]. In this paper, we are interested in obtaining HEE for Einstein- $\Lambda$  in gravity's rainbow and its generalization to  $d$ -dimensional case.

The outline of the paper will be as follows. First, we study a spherical symmetric metric and extract the HEE in Einstein- $\Lambda$  gravity's rainbow for  $(3 + 1)$ -dimensions. Next, we obtain a global equation of hydrostatic equilibrium for compact stars in the higher dimensions in this gravity. The maximum mass for neutron star in the presence and absence of cosmological constant in gravity's rainbow is investigated. In the next section, we compare results obtained for theory with observational compact object. The last section is devoted to closing remarks.

## II. $(3 + 1)$ -DIMENSIONAL HEE IN GRAVITY'S RAINBOW

Here, we present the Einstein gravity with the cosmological constant. The action of this gravity is given by

$$I_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^d x \sqrt{-g} \{R - 2\Lambda\} + I_{Matt}, \quad (1)$$

where  $R$  is the Ricci scalar,  $\Lambda$  is the cosmological constant and  $I_{Matt}$  is the action of matter field. Varying the action (1) with respect to the metric tensor  $g_\mu^\nu$ , the equation of motion for this gravity can be written as

$$G_\mu^\nu + \Lambda g_\mu^\nu = K T_\mu^\nu, \quad (2)$$

where  $K = \frac{8\pi G}{c^4}$  and also,  $G_\mu^\nu$  and  $T_\mu^\nu$  are the Einstein and energy-momentum tensors, respectively.

Here, we intend to obtain the static solutions of Eq. (2) in gravity's rainbow. To do so, we use the metric which is energy dependent. As it was pointed out, one of the basics for gravity's rainbow is deformation of the standard energy-momentum relation

$$E^2 L^2(\varepsilon) - p^2 H^2(\varepsilon) = m^2, \quad (3)$$

in which  $\varepsilon = \frac{E}{E_P}$  where  $E_P$  is the Planck energy. Considering the mentioned upper limit for energies that a particle can obtain, we have

$$\varepsilon \leq 1. \quad (4)$$

The functions  $L(\varepsilon)$  and  $H(\varepsilon)$  are rainbow functions which satisfy the following conditions,

$$\lim_{\varepsilon \rightarrow 0} L(\varepsilon) = 1, \quad \lim_{\varepsilon \rightarrow 0} H(\varepsilon) = 1. \quad (5)$$

These conditions ensure the standard energy-momentum relation in the infrared limit. Now, we are in a position to construct the energy dependent metric as follows [11]

$$h(\varepsilon) = \eta^{\mu\nu} e_\mu(\varepsilon) \otimes e_\nu(\varepsilon), \quad (6)$$

where

$$e_0(\varepsilon) = \frac{1}{L(\varepsilon)} e_0^\sim, \quad e_i(\varepsilon) = \frac{1}{H(\varepsilon)} e_i^\sim, \quad (7)$$

in which the tilde quantities refer to the energy independent frame fields. Therefore, one can assume a spherical symmetric space-time in the following form,

$$ds^2 = \frac{f(r)}{L^2(\varepsilon)} dt^2 - \frac{1}{H^2(\varepsilon)} \left( \frac{dr^2}{g(r)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right), \quad (8)$$

where  $f(r)$  and  $g(r)$  are radial dependent functions which should be determined. In addition,  $L(\varepsilon)$  and  $H(\varepsilon)$  are the energy functions which will be introduced later.

The energy-momentum tensor for a perfect fluid is

$$T^{\mu\nu} = (c^2 \rho + P) U^\mu U^\nu - P g^{\mu\nu}, \quad (9)$$

where  $\rho$  and  $P$  are density and pressure of the fluid which are measured by local observer, respectively, and  $U^\mu$  is defined as

$$U^\mu = \left( \frac{L(\varepsilon)}{\sqrt{f(r)}}, 0, 0, 0 \right), \quad (10)$$

which is the fluid four-velocity with following restriction

$$g_{\mu\nu} U^\mu U^\nu = 1. \quad (11)$$

Using Eq. (9) and the metric introduced in Eq. (8), we can obtain the components of energy-momentum tensor for (3+1)-dimensions as follows

$$T_0^0 = \rho c^2 \quad \& \quad T_1^1 = T_2^2 = T_3^3 = -P. \quad (12)$$

We consider the metric (8) and Eq. (12) for perfect fluid and obtain the components of Eq. (2) with the following forms

$$K c^2 r^2 \rho = \Lambda r^2 + (1 - g) H^2(\varepsilon) - r g' H^2(\varepsilon), \quad (13)$$

$$K r^2 f P = -\Lambda r^2 f - (1 - g) H^2(\varepsilon) f + r g H^2(\varepsilon) f', \quad (14)$$

$$4 K r f^2 P = -4 \Lambda r f^2 + 2 (g f)' H^2(\varepsilon) f + r [g' f' + 2 g f''] H^2(\varepsilon) f - r g H^2(\varepsilon) f'^2, \quad (15)$$

where  $f$ ,  $g$ ,  $\rho$  and  $P$  are functions of  $r$ . We note that the prime and double prime denote the first and second derivatives with respect to  $r$ , respectively.

Using Eqs. (13-15) and after some calculations, we have

$$\frac{dP}{dr} + \frac{(c^2\rho + P)f'}{2f} = 0. \quad (16)$$

Now, we obtain  $f'$  from Eq. (14) as follow

$$f' = \frac{[r^2(\Lambda + KP) + (1 - g)H^2(\varepsilon)]f}{rgH^2(\varepsilon)}. \quad (17)$$

Then, for obtaining  $g$ , we use Eq. (13)

$$g = 1 + \frac{\Lambda}{3H^2(\varepsilon)}r^2 - \frac{c^2KM_{eff}}{4\pi r}, \quad (18)$$

where  $M_{eff}$  is a function of  $r$  and  $\varepsilon$ :  $M_{eff}(r, \varepsilon) = \int \frac{4\pi r^2 \rho(r)}{H^2(\varepsilon)} dr$ . It is notable that the obtained relation for effective mass is a function of radial coordinate and energy functions. This is a direct contribution of the gravity's rainbow. By inserting Eqs. (17) and (18) in Eq. (16), we can extract the HEE in Einstein- $\Lambda$  gravity's rainbow for (3+1)-dimension as

$$\frac{dP}{dr} = \frac{[3c^2GM_{eff}H^2(\varepsilon) + r^3(\Lambda c^4 + 12\pi GP)](c^2\rho + P)}{c^2r[6GM_{eff}H^2(\varepsilon) - c^2r(\Lambda r^2 + 3H^2(\varepsilon))]} \quad (19)$$

We see that this equation is different from HEE, and for  $\Lambda = 0$  and  $H(\varepsilon) = 1$ , it yields the usual TOV equation [44–46]. Also, for  $\Lambda \neq 0$  and  $H(\varepsilon) = 1$ , it reduces to HEE obtained in Ref. [77].

Considering different phenomenologies, one can employ three different cases for rainbow functions.

Motivated by studies conducted in loop quantum gravity and non-commutative geometry, the first case has the following energy functions, [78, 79]

$$L(\varepsilon) = 1 \quad \& \quad H(\varepsilon) = \sqrt{1 - \eta\varepsilon^n}. \quad (20)$$

Using Eqs. (19) and (20), the HEE is

$$\frac{dP}{dr} = \frac{[3c^2GM_{eff}[1 - \eta\varepsilon^n] + r^3(\Lambda c^4 + 12\pi GP)](c^2\rho + P)}{c^2r[6GM_{eff}[1 - \eta\varepsilon^n] - c^2r(\Lambda r^2 + 3[1 - \eta\varepsilon^n])]}, \quad (21)$$

where for  $\eta = 0$ , Eq. (21) reduces to the HEE obtained for Einstein- $\Lambda$  gravity in Ref. [77]. **Here, since  $H(\varepsilon)$  is a square root function, one can find following condition regarding its parameters**

$$\eta < \frac{1}{\varepsilon^n}.$$

The second case is related to the hard spectra from gamma-ray bursts which has energy functions as [13]

$$L(\varepsilon) = \frac{e^{\beta\varepsilon} - 1}{\beta\varepsilon} \quad \& \quad H(\varepsilon) = 1. \quad (22)$$

It is notable that due to the structure of obtained TOV equation (Eq. (19)), this case of energy functions will lead to absence of the effects of gravity's rainbow. In other words, for this case, the TOV equation is independent of energy functions which is not of our interest.

The third case of energy functions is due to consideration of constancy of the velocity of light which leads to the following relation, [80]

$$L(\varepsilon) = H(\varepsilon) = \frac{1}{1 - \lambda\varepsilon}. \quad (23)$$

We use Eq. (23) for obtaining the HEE, so we have

$$\frac{dP}{dr} = \frac{\left[3c^2GM_{eff}\left(\frac{1}{1-\lambda\varepsilon}\right)^2 + r^3(\Lambda c^4 + 12\pi GP)\right](c^2\rho + P)}{c^2r\left[6GM_{eff}\left(\frac{1}{1-\lambda\varepsilon}\right)^2 - c^2r\left(\Lambda r^2 + 3\left(\frac{1}{1-\lambda\varepsilon}\right)^2\right)\right]}, \quad (24)$$

where for  $\lambda = 0$ , Eq. (24) reduces to Einstein- $\Lambda$  gravity [77]. Since the energy functions must be positive valued (for avoiding the change of metric signature), one can extract following condition for this model of gravity's rainbow

$$\lambda < \frac{1}{\varepsilon}.$$

### III. HEE OF GRAVITY'S RAINBOW IN HIGHER DIMENSIONS

Now, we are going to obtain the HEE in Einstein- $\Lambda$  gravity's rainbow for higher dimensions ( $d \geq 5$  where  $d$  represents the dimensionality of space-time). We consider the following metric

$$ds^2 = \frac{f(r)}{L^2(\varepsilon)} dt^2 - \frac{1}{H^2(\varepsilon)} \left( \frac{dr^2}{g(r)} + r^2 d\Omega_k^2 \right), \quad (25)$$

where

$$d\Omega_k^2 = d\theta_1^2 + \sum_{i=2}^{d-2} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2. \quad (26)$$

Then, we must obtain Eq. (12) for arbitrarily dimensions, therefore we have,

$$T_0^0 = c^2 \rho \quad \& \quad T_1^1 = T_2^2 = T_3^3 = \dots = T_{d-1}^{d-1} = -P. \quad (27)$$

We use the above equation to obtain a global relation for the HEE in higher dimensions for Einstein- $\Lambda$  gravity's rainbow. Using Eqs. (2) and (27) for the metric (25), the components of the equation of motion for Einstein- $\Lambda$  gravity's rainbow can be written as

$$K_d c^2 r^2 \rho = \Lambda r^2 + \frac{(d-2)(d-3)}{2} (1-g) H^2(\varepsilon) - \frac{(d-2)}{2} r g' H^2(\varepsilon), \quad (28)$$

$$K_d r^2 f P = -\Lambda r^2 f - \frac{(d-2)(d-3)}{2} (1-g) H^2(\varepsilon) f + \frac{(d-2)}{2} r g H^2(\varepsilon) f', \quad (29)$$

$$\begin{aligned} 4K_d r f^2 P = & -4\Lambda r f^2 - \frac{2(d-3)(d-4)}{r} (1-g) H^2(\varepsilon) f^2 + r [g' f' + 2g f''] H^2(\varepsilon) f \\ & + 2(d-3) (g f)' H^2(\varepsilon) f - r g H^2(\varepsilon) f'^2, \end{aligned} \quad (30)$$

in which  $K_d = \frac{8\pi G_d}{c^4}$  and  $G_d$  is the gravitational constant in  $d$ -dimensions and  $G_d$  is defined as  $G_d = G V_{d-4}$ , where  $G$  denotes the four dimensional gravitational constant and also,  $V_{d-4}$  is the volume of extra space. We obtain Eq. (16) in the same manner as it was described for  $(3+1)$ -dimensions, then using Eq. (29), we get  $f'$  as follows

$$f' = \frac{2 \left[ r^2 (\Lambda + K_d P) + \frac{(d-2)(d-3)}{2} (1-g) H^2(\varepsilon) \right] f}{r g (d-2) H^2(\varepsilon)}. \quad (31)$$

We calculate  $g$  of Eq. (28) as

$$g = 1 + \frac{2\Lambda}{(d-1)(d-2)H^2(\varepsilon)} r^2 - \frac{c^2 K_d M_{eff}(r, \varepsilon) \Gamma\left(\frac{d-1}{2}\right)}{(d-2) \pi^{(d-1)/2} r^{d-3}}. \quad (32)$$

It should be noted that we have used  $M_{eff}(r, \varepsilon) = \int \frac{2\pi^{(d-1)/2}}{\Gamma((d-1)/2) H^2(\varepsilon)} r^{d-2} \rho(r) dr$  in the above equation and also  $\Gamma$  is the gamma function, which satisfies some conditions as  $\Gamma(1/2) = \sqrt{\pi}$ ,  $\Gamma(1) = 1$  and  $\Gamma(x+1) = x\Gamma(x)$ .

Using Eqs. (31) and (32) in Eq. (16), we can get the HEE in the Einstein- $\Lambda$  gravity for  $d$ -dimensions

$$\frac{dP}{dr} = \frac{\left[ \frac{(d-1)(d-3)\Gamma(\frac{d-1}{2})c^2 K_d M_{eff} H^2(\varepsilon)}{4\pi^{(d-1)/2} r^{d-1}} + \left( \Lambda + \frac{(d-1)K_d P}{2} \right) \right] (c^2 \rho + P)}{r \left[ -\Lambda + \frac{(d-1)}{r^{d-1}} \left( \frac{\Gamma(\frac{d-1}{2})c^2 K_d M_{eff}}{2\pi^{(d-1)/2}} - \frac{(d-2)r^{d-3}}{2} \right) H^2(\varepsilon) \right]}, \quad (33)$$

where for  $(3+1)$ -dimensional limit, Eq. (33) reduces to Eq. (19).

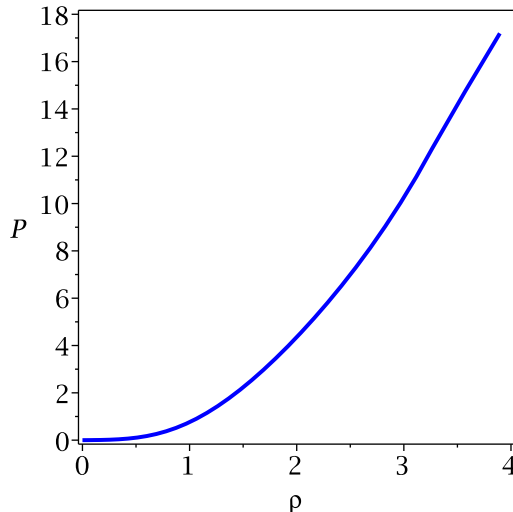


FIG. 1: Equation of state of neutron star matter (pressure,  $P$  ( $10^{35}$  erg/cm<sup>3</sup>) versus density,  $\rho$  ( $10^{15}$  gr/cm<sup>3</sup>)).

#### IV. STRUCTURE PROPERTIES OF NEUTRON STAR

##### A. Equation of state of neutron star matter

It is stated that one can derive the properties of neutron star matter by obtaining the equation of state of neutron star matter. The constituents of the interior part of a neutron star include the neutrons, protons, electrons and muons which are in charge neutrality and beta equilibrium conditions (beta-stable matter) [81]. On the other hand, for studying the equation of state of stars, there are several approaches which among them one can name the microscopic constrained variational calculations based on the cluster expansion of the energy functional [82, 83]. In this approach, the new Argonne AV18 and charged dependent Reid-93 are employed as the two-nucleon potentials (see Refs. [84, 85], for more details). Also, a good convergence, lack of need for any free parameter in formalism and more accuracy comparing to other semi-empirical parabolic approximation methods are of the advantages of these methods. The modern nucleon-nucleon potentials are isospin projection ( $Tz$ ) dependent in which a microscopic computation of asymmetry energy is carried on for the asymmetric nuclear matter calculations (see Ref. [47] for more details).

In this paper, we employ AV18 potential [82, 83] for calculating the modern equation of state for neutron star matter. Then, we will study some physical properties of neutron star structure. Our result for obtained equation of state of neutron star matter is presented in Fig. 1. We extract the mathematical forms for the equation of state presented in Fig. 1 as

$$P = \sum_{i=1}^7 \mathcal{A}_i \rho^{7-i}, \quad (34)$$

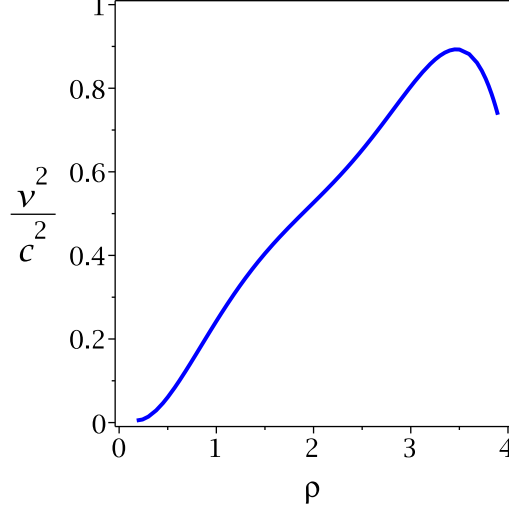
where  $\mathcal{A}_i$  are

$$\begin{aligned} \mathcal{A}_1 &= -3.518 \times 10^{-57}, \\ \mathcal{A}_2 &= 3.946 \times 10^{-41}, \\ \mathcal{A}_3 &= -1.67 \times 10^{-25}, \\ \mathcal{A}_4 &= 3.242 \times 10^{-10}, \\ \mathcal{A}_5 &= -1.458 \times 10^5, \\ \mathcal{A}_6 &= 2.911 \times 10^{19}, \\ \mathcal{A}_7 &= -9.983 \times 10^{31}. \end{aligned}$$

In order to have a better insight regarding the properties of these neutron stars, we study both energy and stability conditions in the following subsections.

TABLE I: Energy conditions for neutron star.

$\rho_0(10^9 \frac{kg}{cm^3})$	$P_0(10^9 \frac{kg}{cm^3})$	NEC	WEC	SEC	DEC
3895.99	1910.39	✓	✓	✓	✓

FIG. 2: Sound speed ( $v^2/c^2$ ) versus density ( $\rho \times 10^{15}$  (gr/cm<sup>3</sup>)).

### 1. Energy conditions

Here, we investigate the energy conditions such as the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and also, dominant energy condition (DEC) at the center of neutron star. We have

$$NEC \rightarrow P_c + \rho_c \geq 0, \quad (35)$$

$$WEC \rightarrow P_c + \rho_c \geq 0, \quad \& \quad \rho_c \geq 0, \quad (36)$$

$$SEC \rightarrow P_c + \rho_c \geq 0, \quad \& \quad 3P_c + \rho_c \geq 0, \quad (37)$$

$$DEC \rightarrow \rho_c > |P_c|, \quad (38)$$

where  $\rho_c$  and  $P_c$  are the density and pressure at the center of neutron star ( $r = 0$ ), respectively. Considering Fig. 1 and the above conditions (35-38), our results are given in table I. According to Fig. 1 and table I, we observe that all energy conditions are satisfied. So, the equation of state of neutron star matter introduced in this paper is suitable.

### 2. Stability

In order to investigate the stability of equation of state of neutron star matter for a physically acceptable model, one expects that the velocity of sound ( $v$ ) be less than the light's velocity ( $c$ ) [86, 87]. By considering the stability condition ( $0 \leq v^2 = \left(\frac{dP}{d\rho}\right) \leq c^2$ ) and Eq. (34), and comparing them with diagrams related to speed of sound-density relationship plot in Fig. 2, it is evident that this equation of state of neutron star matter satisfies the inequality  $0 \leq v^2 \leq c^2$ .

Our investigations show that our equation of state of neutron star matter satisfies both energy and stability conditions. Now, we focus on investigation of gravitational mass and radius for neutron stars in gravity's rainbow.

TABLE II: Structure properties of neutron star for different values of  $H(\varepsilon)$  ( $1.2 \leq H(\varepsilon) \leq 1.67$ ).

$H(\varepsilon)$	$M_{max} (M_{\odot})$	$R (km)$	$R_{Sch} (km)$	$\bar{\rho} (10^{15} g cm^{-3})$	$\sigma (10^{-1})$	$z (10^{-1})$
1.70	2.87	14.32	8.46	0.46	5.91	5.63
1.67	2.81	14.00	8.28	0.49	5.91	5.65
1.60	2.69	13.47	7.93	0.52	5.89	5.59
1.50	2.52	12.63	7.43	0.59	5.88	5.58
1.40	2.37	11.79	6.99	0.69	5.93	5.66
1.30	2.19	10.95	6.46	0.79	5.90	5.61
1.20	2.02	10.10	5.95	0.93	5.89	5.61

### B. Properties of neutron stars in gravity's rainbow without the cosmological constant

Study of compact objects such as neutron stars has been of great interest for the astrophysicists in the last two decades (see Refs. [88–95] for more details). Of the most important features of neutron stars are the maximum mass and radius, and determining them. Because, there is a critical maximum mass in which for masses smaller than that the degeneracy pressure originated from the nucleons prevents an object from becoming a black hole [81]. Therefore, obtaining the maximum gravitational mass of neutron stars is of great importance in astrophysics. Due to many errors in direct ways of measuring mass and radius of the neutron stars by observations of the X-ray pulsars and X-ray bursts, one is not able to obtain an accurate mass and radius for these stars. On the other hand, using the binary radio pulsars [96–99], leads to highly accurate results for the mass and radius of neutron stars.

Now, by employing the equation of state of neutron star matter which is presented in Fig. 1, and numerical approach for integrating the HEE obtained in the equation (19), we obtain the maximum mass and radius of neutron star. It is worthwhile to mention that the neutron star mass and radius are central mass density ( $\rho_c$ ) dependent in this approach. For this purpose, one can consider the boundary conditions  $P(r=0) = P_c$  and  $m(r=0) = 0$ , and integrates Eq. (19) outwards to a radius  $r = R$  in which  $P$  vanishes for selecting a  $\rho_c$ . This leads to the neutron star radius  $R$  and mass  $M = m(R)$ . The results are presented in various tables and figures.

The Einstein gravity case has been investigated in Ref. [100], and the maximum mass of neutron stars by using the modern equations of state of neutron star matter derived from microscopic calculations was obtained. It was shown that the maximum mass for neutron stars is about  $1.68M_{\odot}$  in Einstein gravity. The effect of the cosmological constant on maximum mass of neutron star was investigated, and it was pointed out that considering the positive values of the cosmological constant, the maximum mass of neutron star decreases [77] and also, the behavior of neutron star (for example; diagram related to the maximum mass-radius) for the negative values of the cosmological constant are not logical (see Ref. [77] for more details).

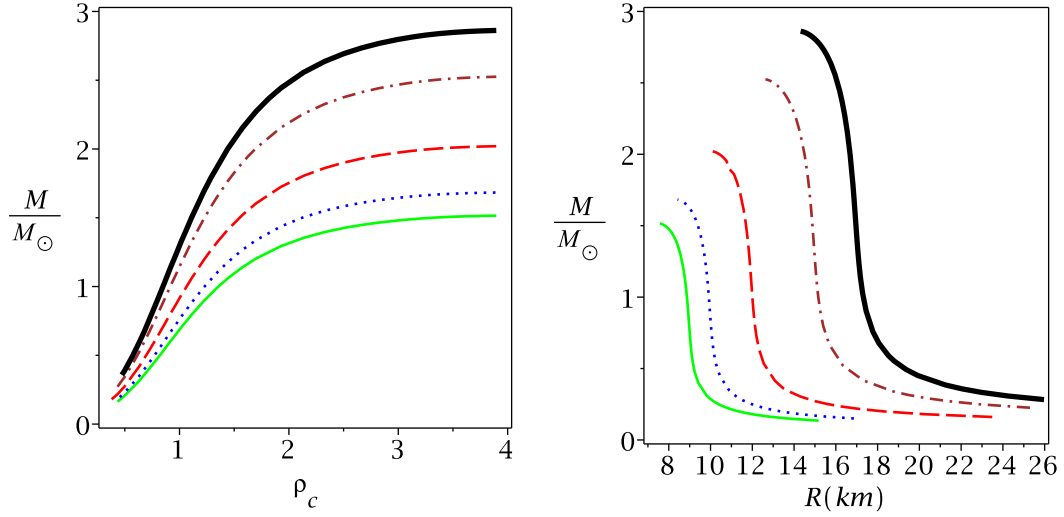
On the other hand, the maximum mass of neutron star is still an open question. There are some observational evidences which indicate that the maximum mass of neutron stars can be more than  $1.68M_{\odot}$ . For example, Jacoby et al [101] and Verbiest et al [102] for a binary system used the detection of Shapiro delay to measure the masses of both the neutron star and its binary companion. Also, using the same approach, Demorest et al [103] has performed radio timing observations for the binary millisecond pulsar PSR J1614-2230 where the measured mass for this pulsar was obtained about  $1.97M_{\odot}$ . The mass of other compact objects were obtained about  $1.8M_{\odot}$  for Vela X-1 [104], PSR J0348+0432 [105] about  $2.01M_{\odot}$ , 4U 1700-377 [106] about  $2.4M_{\odot}$ , and J1748-2021B [107] about  $2.7M_{\odot}$ . Also, A. W. Steiner et al determined an empirical dense matter equation of state from a heterogeneous data set of six neutron stars. They also showed that, the radius of a neutron star must be in the range  $R \leq (11 \sim 14) km$  [108]. In the present paper, we would like to see whether the gravity's rainbow and obtained maximum mass for neutron stars by employing the modern equations of state of neutron star matter derived from microscopic calculations can predict existence of the maximum mass and radius of neutron stars more than  $1.68M_{\odot}$  and less than  $14km$ , respectively. Therefore, we consider the neutron stars with radius less than  $R \leq 14km$  and separate the maximum mass for  $M_{max} > 2M_{\odot}$  and  $M_{max} < 2M_{\odot}$  in tables II and III, respectively.

It is notable that, for  $H(\varepsilon) = 1$ , the maximum mass of neutron star reduces to the result that was obtained in Einstein gravity [77, 100], as expected (because, this case ( $H(\varepsilon) = 1$ ) is denoted as the absence of gravity's rainbow. In other words, in this case, the effects of the gravity's rainbow are vanished). On the other hand, our results show that, by increasing rainbow function more than 1 ( $H(\varepsilon) > 1$ ), the maximum mass of neutron star increases (see tables II and III). In other words, our results cover the mass measurement of massive neutron stars, and also, predict that



TABLE III: Structure properties of neutron star for different values of  $H(\varepsilon)$  ( $0.5 \leq H(\varepsilon) \leq 1.1$ ).

$H(\varepsilon)$	$M_{max} (M_\odot)$	$R (km)$	$R_{Sch} (km)$	$\bar{\rho} (10^{15} g cm^{-3})$	$\sigma (10^{-1})$	$z (10^{-1})$
1.10	1.85	9.26	5.45	1.11	5.88	5.59
1.00	1.68	8.42	4.95	1.34	5.88	5.58
0.90	1.51	7.58	4.45	1.65	5.87	5.56
0.80	1.35	6.73	3.98	2.10	5.91	5.64
0.70	1.18	5.89	3.48	2.74	5.91	5.62
0.60	1.01	5.05	2.98	3.72	5.90	5.61
0.50	0.84	4.21	2.48	5.35	5.89	5.58

FIG. 3: Gravitational mass versus central mass density (left diagram) and radius (right diagram), for  $H(\varepsilon) = 0.9$  (continuous line),  $H(\varepsilon) = 1$  (dotted line),  $H(\varepsilon) = 1.2$  (dashed line),  $H(\varepsilon) = 1.5$  (dashed-dotted line) and  $H(\varepsilon) = 1.7$  (bold line).

the mass of neutron stars in gravity's rainbow can be in the range larger than  $1.68M_\odot$  (see tables II and III for more details). In order to conduct further investigations, we plot diagram related to the gravitational mass versus central mass density (or radius) in Fig. 3. The variation of the gravitational mass versus different values of the rainbow function is presented in this figure. On the contrary, for  $H(\varepsilon) < 1$ , both maximum mass of neutron stars and radius of these stars are smaller than the Einstein case (see table III and Fig. 3). This emphasizes the contributions of the gravity's rainbow on properties of neutron stars.

In the above tables, we used the obtained TOV equations in Eqs. (21) and (24) for  $H(\varepsilon) < 1$  and  $H(\varepsilon) > 1$ , respectively, to extract the structure of neutron stars. As one can see, due to the structure of obtained TOV equation (Eq. (19)), for case (22), the obtained TOV equation is independent of energy functions which is not of our interest.

In the following, we are going to investigate other properties of neutron star in the gravity's rainbow.

### 1. Schwarzschild radius

Using Eq. (18), and this fact that  $g(r = R_{Sch}) = 0$ , we calculate Schwarzschild radius for the obtained masses in gravity's rainbow as

$$R_{Sch} = \frac{2GM_{eff}}{c^2}. \quad (39)$$

As it was pointed out, the mass of neutron star depends on rainbow function and thus we expect the Schwarzschild

radius varies depending on choices of rainbow function. The results are presented in tables II and III. It is evident that, the Schwarzschild radius is an increasing function of rainbow function.

## 2. Average density

The average density of a neutron star has the following form

$$\bar{\rho} = \frac{3M_{eff}}{4\pi R^3}, \quad (40)$$

where it is a decreasing function of  $H(\varepsilon)$ .

Obtained central density by using the mentioned equation of state of neutron star matter is about  $3.9 \times 10^{15} g cm^{-3}$  and this density is larger than the normal nuclear density,  $\rho_0 = 2.7 \times 10^{14} g cm^{-3}$  [109]. On the other hand, by considering the rainbow function less than 0.6 ( $H(\varepsilon) < 0.6$ ), obtained average density for neutron stars is larger than the central density (see the last row of table III). In other words, when the rainbow function is larger than 0.6 ( $H(\varepsilon) > 0.6$ ), the neutron star is compatible.

## 3. Compactness

The compactness of a spherical object may be defined as the ratio of Schwarzschild radius to the radius of object ( $\sigma = R_{Sch}/R$ ), which may be indicated as the strength of gravity. Obtained  $\sigma$  in this gravity shows that the strength of gravity is almost the same for these neutron stars.

## 4. Gravitational redshift

Using Eq. (18), we can obtain the gravitational redshift in the following form

$$z = \frac{1}{\sqrt{1 - \frac{2GM_{eff}}{c^2 R}}} - 1, \quad (41)$$

where  $M_{eff}$  is the mass of neutron stars in which is rainbow function dependant. This relation indicates that to have a physical redshift, the radius of a neutron star should be greater than the Schwarzschild radius,  $R > \frac{2GM_{eff}}{c^2} = R_{Sch}$ . The results related to the gravitational redshift shows that for neutron stars with different mass, this quantity is almost the same (see the last column of tables II and III). We plot the diagrams related to the gravitational redshift versus mass in Fig. 4. This figure shows that, for different values of rainbow function, this quantity is not affected considerably.

## 5. Dynamical stability

The dynamical stability of the stellar model against the infinitesimal radial adiabatic perturbation was introduced by Chandrasekhar [110]. Then, this stability condition was developed and applied to astrophysical cases by many authors [111–114]. The adiabatic index ( $\gamma$ ) is defined in following form

$$\gamma = \frac{\rho c^2 + P}{c^2 P} \frac{dP}{d\rho}. \quad (42)$$

In order to investigate the dynamical stability condition,  $\gamma$  should be more than  $\frac{4}{3}$  ( $\gamma > \frac{4}{3} = 1.33$ ) everywhere within the isotropic star. For this purpose, we plot two diagrams related to  $\gamma$  versus radius for different values of rainbow functions in Figs. 5 and 6. As one can see, this stellar model is stable against the radial adiabatic infinitesimal perturbations.

Also, we plot the pressure (density) versus distance from the center of neutron star. As one can see, the pressure and density are maximum at the center and decreases monotonically towards the boundary (see Figs 7 and 8).

As one can see, considering the radius of neutron stars less than  $14 km$  (according to the work of A. W. Steiner et al. [108]) and by using the average density, we obtained an upper and a lower limitations for rainbow function.

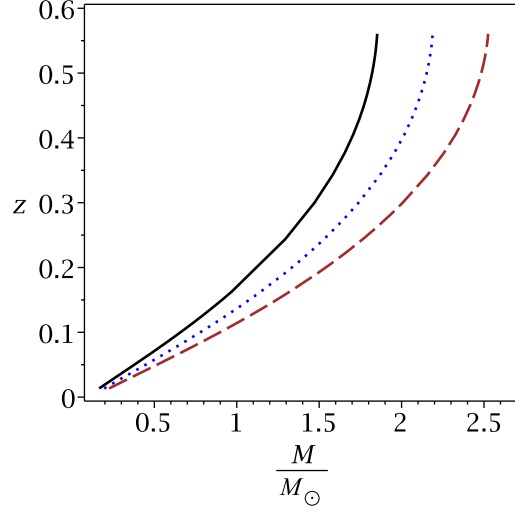


FIG. 4: The gravitational redshift versus mass for  $H(\varepsilon) = 1.1$  (continuous line),  $H(\varepsilon) = 1.3$  (doted line) and  $H(\varepsilon) = 1.5$  (dashed line).

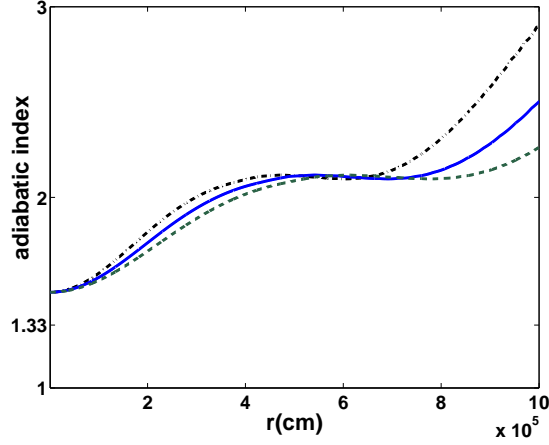


FIG. 5: Adiabatic index versus radius for  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.4$  (dashed-dotted line).

Therefore, the obtained solutions in gravity's rainbow are valid in the range of  $0.6 < H(\varepsilon) \leq 1.67$ . In other words, gravity's rainbow cover all neutron stars and pulsars which have the maximum mass less than  $2.81M_\odot$  (see tables II and III). Indeed, these results cover wide range of neutron stars and pulsars such as Vela X-1, PSR J0348+0432, 4U 1700-377, J1748-2021B, PSR J1903 + 327, Cen X - 3, PSR B1913 + 16, PSR J0737 - 3039, PSR J0737 - 3039B and SMC X - 1.

### C. Relation between the mass of neutron stars and Planck mass

Here we derive the relation between neutron star mass and the Planck mass in the gravity's rainbow according to Burrows and Ostriker's work [115]. We know that in addition to the degenerate pressure of nucleons, the pressure due to the strong repulsive inter-nucleons force balances the pressure due to the gravitation force in the neutron stars. Here, the nucleon-nucleon interaction which is so strong, is taken place through the pion exchange. Using this idea, we can consider the average density of a neutron star in term of the nucleus density using the following relation

$$\rho_{nuc} \sim \frac{3m_p}{4\pi\lambda_\pi^3}, \quad (43)$$

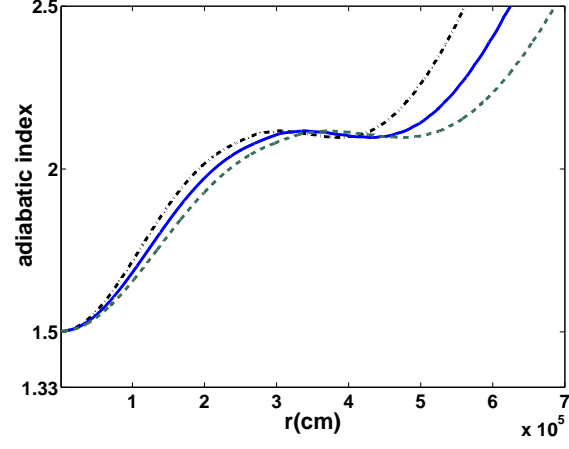


FIG. 6: Adiabatic index versus radius for  $H(\varepsilon) = 1.1$  (dashed line),  $H(\varepsilon) = 1.0$  (continuous line) and  $H(\varepsilon) = 0.9$  (dashed-dotted line).

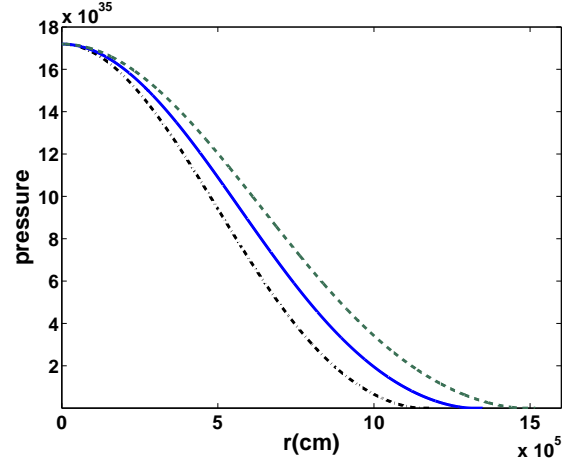


FIG. 7: Pressure ( $\text{erg}/\text{cm}^3$ ) versus radius for  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.4$  (dashed-dotted line).

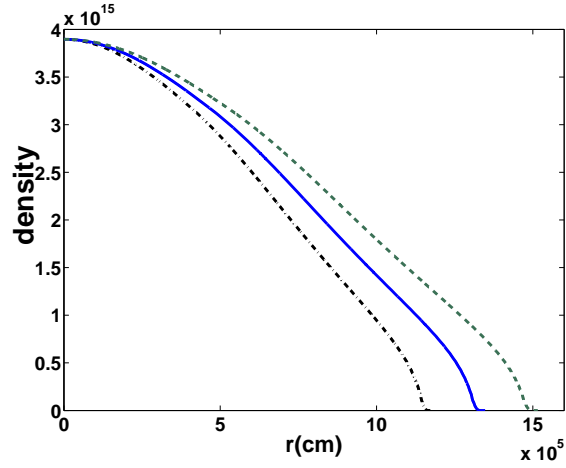


FIG. 8: Density ( $\text{gr}/\text{cm}^3$ ) versus radius for  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.4$  (dashed-dotted line).

where  $m_p$  is the proton mass and  $\lambda_\pi = \frac{\hbar}{m_\pi c}$  is the Compton wavelength. Here  $m_\pi$  is the pion mass. On the other hand, the mass of a neutron star is set by a general relativity instability. The condition for Einstein gravity instability is  $R = \frac{2GM}{c^2}$  (see Ref. [115] for more details). Now, we use this idea for gravity's rainbow. The Schwarzschild radius and the average density of a neutron star in gravity's rainbow are obtained by equations (39) and (40) as follows

$$R_{Sch} = \frac{2GM_{eff}}{c^2} = \frac{2GM}{c^2 H^2(\varepsilon)}, \quad (44)$$

$$\bar{\rho} = \frac{3M_{eff}}{4\pi R^3} = \frac{3M}{4\pi H^2(\varepsilon) R^3}, \quad (45)$$

where

$$M_{eff}(r, \varepsilon) = \frac{1}{H^2(\varepsilon)} \int 4\pi r^2 \rho(r) dr. \quad (46)$$

Therefore, we have  $M_{eff} = \frac{M}{H^2(\varepsilon)}$ . Now, we can derive the corresponding mass using the equations (44) and (45) and considering  $\rho_{nuc}$  as follows

$$\begin{aligned} M &\sim H^2(\varepsilon) \left( \frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_p^2} \left( \frac{\eta_\pi}{2\eta_p} \right)^{3/2} \\ &\sim H^2(\varepsilon) M_{Ch} \left( \frac{\eta_\pi}{2\eta_p} \right)^{3/2} \\ &\sim H^2(\varepsilon) m_{pl} \eta_p^2 \left( \frac{\eta_\pi}{2\eta_p} \right)^{3/2}, \end{aligned} \quad (47)$$

where  $M_{Ch}$  is the Chandrasekhar mass. In Eq. (47),  $M_{Ch} \sim \left( \frac{\hbar c}{G} \right)^{3/2} \frac{1}{m_p^2} \sim m_{pl} \eta_p^2$ ,  $\eta_p = \frac{m_{pl}}{m_p}$  and  $\eta_\pi = \frac{m_{pl}}{m_\pi}$  where  $m_{pl}$  is the Planck mass (see Ref. [115] for more details). Therefore, the relation between the neutron star mass and the Planck mass is obtained by above equation. It is notable that for  $H(\varepsilon) = 1$ , Eq. (47) reduces to Einstein gravity [115]. On the other hand, we can rewrite the equation (47) in the following form

$$M \sim H^2(\varepsilon) \Psi_{EN}, \quad (48)$$

where  $\Psi_{EN} = m_{pl} \eta_p^2 \left( \frac{\eta_\pi}{2\eta_p} \right)^{3/2}$ .

#### D. Properties of neutron stars in gravity's rainbow with cosmological constant

In this subsection, we take the effects of variation of the cosmological constant into account. In other words, the effects of variation of the cosmological constant on maximum mass and radius of the neutron stars in the presence of Einstein gravity's rainbow are investigated. Our results are presented in table IV.

As one can see, interestingly, the maximum mass of neutron star is a decreasing function of the cosmological constant (see table IV for more details). For further investigations of the behavior of the mass versus radius, we have plotted the following diagrams (Figs. 9 and 10).

It is evident from Figs. 9 and 10 that the behavior of mass as a function of the central mass density (or radius) is highly sensitive to the variation of cosmological constant. For large values of cosmological constant (Fig. 10), the behavior is completely modified. In this case, in the presence of large values of cosmological constant, the behavior of the mass versus radius of neutron star will be modified into a quark like behavior.

It is a matter of calculation to show that the Schwarzschild radius of gravity's rainbow in the presence of the cosmological constant is

$$R_{Sch} = \frac{\left[ H^2(\varepsilon) \left( \frac{3GM_{eff}}{c^2} + \sqrt{\frac{H^2(\varepsilon)}{\Lambda} + \frac{9G^2 M_{eff}^2}{c^4}} \right) \right]^{1/3}}{\Lambda^{1/3}} - \frac{H^{4/3}(\varepsilon)}{\left[ \left( \frac{3GM_{eff}}{c^2} + \sqrt{\frac{H^2(\varepsilon)}{\Lambda} + \frac{9G^2 M_{eff}^2}{c^4}} \right) \Lambda^2 \right]^{1/3}}.$$

TABLE IV: Structure properties of neutron star for  $H(\varepsilon) = 1$  (up table) and  $H(\varepsilon) = 1.5$  (down table), respectively, with different values of  $\Lambda$ .

$\Lambda \text{ (} m^{-2} \text{)}$	$M_{max} \text{ (} M_{\odot} \text{)}$	$R \text{ (} km \text{)}$	$R_{Sch} \text{ (} km \text{)}$	$\bar{\rho} \text{ (} 10^{15} g \text{ cm}^{-3} \text{)}$	$\sigma(10^{-1})$	$z(10^{-1})$
$1.00 \times 10^{-16}$	1.68	8.42	4.95	1.34	5.88	5.58
$1.00 \times 10^{-14}$	1.68	8.42	4.95	1.34	5.88	5.58
$5.00 \times 10^{-14}$	1.68	8.41	4.95	1.33	5.88	5.58
$1.00 \times 10^{-13}$	1.67	8.40	4.92	1.33	5.86	5.54
$5.00 \times 10^{-13}$	1.62	8.34	4.77	1.32	5.72	5.29
$1.00 \times 10^{-12}$	1.56	8.25	4.60	1.31	5.57	5.03
$5.00 \times 10^{-12}$	1.12	7.47	3.30	1.27	4.42	3.38
$1.00 \times 10^{-11}$	0.78	6.65	2.29	1.25	3.46	2.36

$\Lambda \text{ (} m^{-2} \text{)}$	$M_{max} \text{ (} M_{\odot} \text{)}$	$R \text{ (} km \text{)}$	$R_{Sch} \text{ (} km \text{)}$	$\bar{\rho} \text{ (} 10^{14} g \text{ cm}^{-3} \text{)}$	$\sigma(10^{-1})$	$z(10^{-1})$
$1.00 \times 10^{-16}$	2.52	12.63	7.43	5.94	5.88	5.58
$1.00 \times 10^{-14}$	2.52	12.63	7.43	5.94	5.88	5.58
$5.00 \times 10^{-14}$	2.51	12.62	7.40	5.94	5.86	5.54
$1.00 \times 10^{-13}$	2.50	12.61	7.37	5.92	5.85	5.51
$5.00 \times 10^{-13}$	2.44	12.51	7.19	5.91	5.75	5.34
$1.00 \times 10^{-12}$	2.34	12.38	6.90	5.86	5.57	5.03
$5.00 \times 10^{-12}$	1.68	11.21	4.95	5.69	4.41	3.38
$1.00 \times 10^{-11}$	1.17	9.98	3.45	5.58	3.46	2.36

TABLE V: Properties of neutron star with and without the cosmological constant.

$H(\varepsilon)$	$\Lambda \text{ (} m^{-2} \text{)}$	$M_{max} \text{ (} M_{\odot} \text{)}$	$R \text{ (} km \text{)}$	$R_{Sch} \text{ (} km \text{)}$	$\bar{\rho} \text{ (} 10^{15} g \text{ cm}^{-3} \text{)}$	$\sigma(10^{-1})$	$z(10^{-1})$
1.00	$1 \times 10^{-52}$	1.68	8.42	4.95	1.34	5.88	5.58
	0	1.68	8.42	4.95	1.34	5.88	5.58
1.30	$1 \times 10^{-52}$	2.19	10.95	6.46	0.79	5.90	5.61
	0	2.19	10.95	6.46	0.79	5.90	5.61
1.60	$1 \times 10^{-52}$	2.69	13.47	7.93	0.52	5.89	5.59
	0	2.69	13.47	7.93	0.52	5.89	5.59

The above equation shows that the Schwarzschild radius is modified in the gravity's rainbow. In other words, the Schwarzschild radius depends on the rainbow function. It is notable that, when  $H(\varepsilon) = 1$ , the Schwarzschild radius reduces to the obtained Schwarzschild radius in Ref. [77]. The results show that the Schwarzschild radius is a decreasing function of the cosmological constant (see table IV for more details). We present the results of the average density and gravity strength in the presence of the cosmological constant in table IV. We find that increasing the cosmological constant leads to decreasing both the strength of gravity and the maximum mass of neutron star. On the other hand, calculations related to the average density show that by increasing  $\Lambda$ , the average density decreases.

We extract the gravitational redshift in gravity's rainbow and in the presence of cosmological constant in the following form

$$z = \frac{1}{\sqrt{1 + \frac{\Lambda R^2}{3H^2(\varepsilon)} - \frac{2GM_{eff}}{c^2 R}}} - 1. \quad (49)$$

The results show that, the gravitational redshift is a decreasing function of the cosmological constant and an

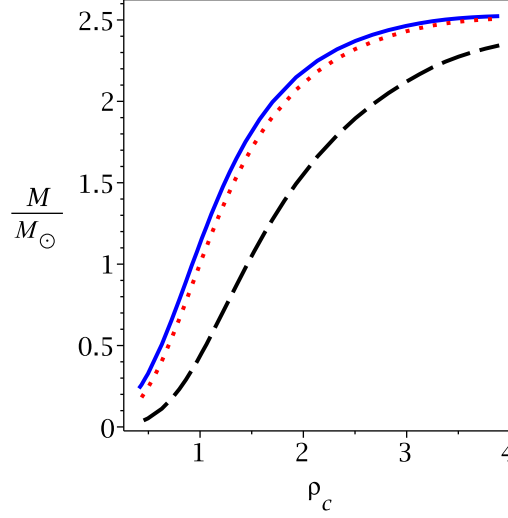


FIG. 9: Gravitational mass of neutron star versus central mass density for  $H(\varepsilon) = 1.5$ ,  $\Lambda = 1 \times 10^{-14}$  (continuous line),  $\Lambda = 1 \times 10^{-13}$  (dotted line) and  $\Lambda = 1 \times 10^{-12}$  (dashed line).

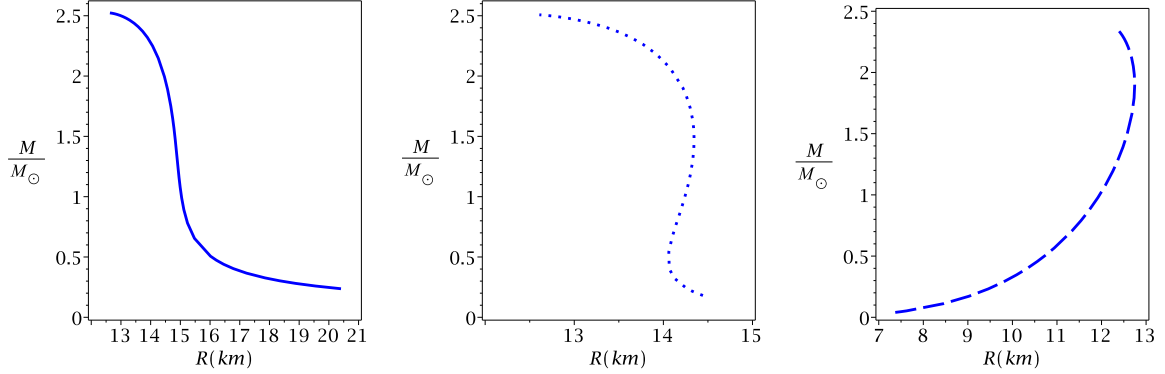


FIG. 10: Gravitational mass of neutron star versus radius for  $H(\varepsilon) = 1.5$ ,  $\Lambda = 1 \times 10^{-14}$ ,  $\Lambda = 1 \times 10^{-13}$  and  $\Lambda = 1 \times 10^{-12}$ , from left to right, respectively.

increasing function of  $M_{eff}$ . In addition, in order to have a finite and real redshift, we should set  $1 + \frac{\Lambda R^2}{3H^2(\varepsilon)} - \frac{2GM_{eff}}{c^2 R} > 0$  and therefore,  $R > R_{Sch}$ , as we expected.

Also, the dynamical stability for neutron stars in gravity's rainbow and in the presence of cosmological constant shows that these stars have dynamical stability in all over the neutron stars (see Figs. 11 and 12, for more details).

In order to investigate internal structure of the neutron star in more details, we plot the pressure (density) versus distance from the center of neutron star in the presence the cosmological constant in gravity's rainbow. Figures 13 and 14 show that, the pressure and density are maximum at the center and decrease monotonically towards the boundary.

The value of the cosmological constant is an open question. On the other hand, from the perspective of cosmology, this value is about  $10^{-52} m^{-2}$ . As one can see, the cosmological constant has no significant effect when its value is about  $10^{-52} m^{-2}$  (see table V for more details). In order to examine the effects of the cosmological constant on properties of the neutron star, we should consider a toy model in which its value is about less than  $10^{-14} m^{-2}$ . The results show that by decreasing the cosmological constant (less than  $10^{-14} m^{-2}$ ,  $\Lambda < 10^{-14} m^{-2}$ ), the maximum mass and also the radius of this star are not modified. In other words, for  $\Lambda < 10^{-14} m^{-2}$ , the cosmological constant does not affect the maximum and radius of neutron star (see table IV for more details). One of the results of this work is that when the value of the cosmological constant is about  $10^{-52} m^{-2}$ , this constant does not play a role in the structure of neutron stars, but by taking larger values for it, the maximum mass and its radius are reduced.

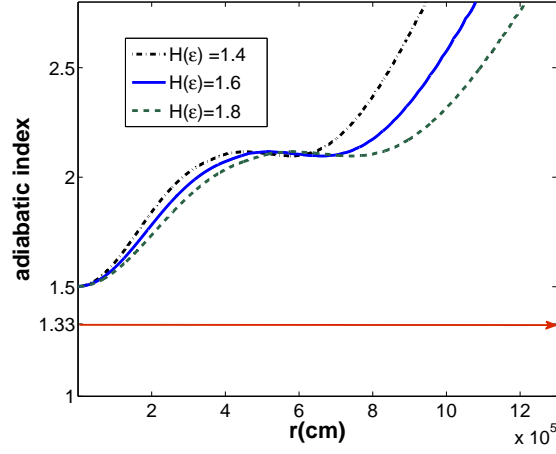


FIG. 11: Adiabatic index versus radius for  $\Lambda = 1 \times 10^{-12}$ ,  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.40$  (dashed-dotted line).

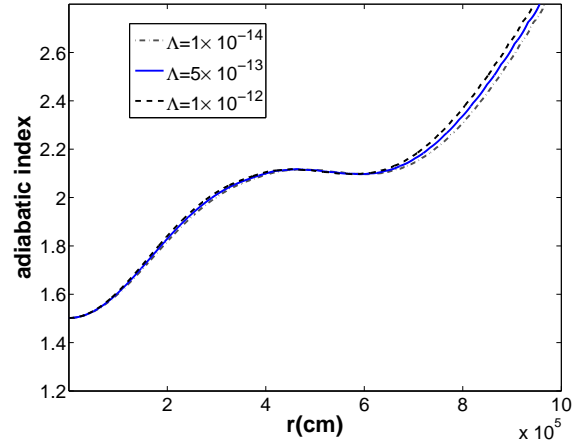


FIG. 12: Adiabatic index versus radius for  $H(\varepsilon) = 1.40$ ,  $\Lambda = 1 \times 10^{-12}$  (dashed line),  $\Lambda = 5 \times 10^{-13}$  (continuous line) and  $\Lambda = 1 \times 10^{-14}$  (dashed-dotted line).

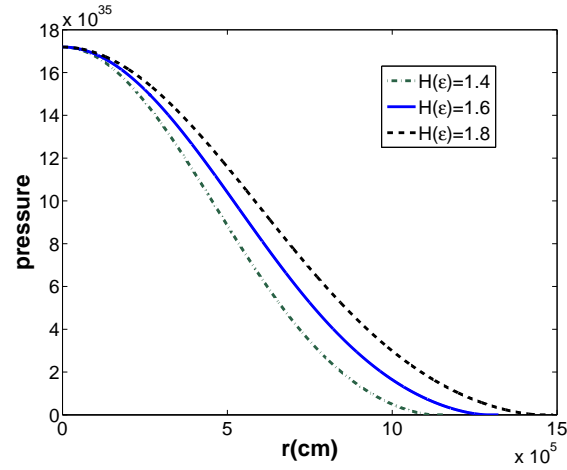


FIG. 13: Pressure ( $\text{erg}/\text{cm}^3$ ) versus radius for  $\Lambda = 1 \times 10^{-12}$ ,  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.40$  (dashed-dotted line).



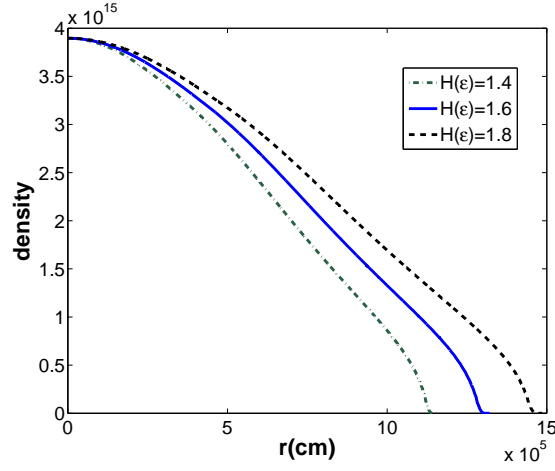


FIG. 14: Density ( $\text{gr}/\text{cm}^3$ ) versus radius for  $\Lambda = 1 \times 10^{-12}$ ,  $H(\varepsilon) = 1.80$  (dashed line),  $H(\varepsilon) = 1.60$  (continuous line) and  $H(\varepsilon) = 1.40$  (dashed-dotted line).

TABLE VI: Properties of neutron star with and without the cosmological constant.

$H(\varepsilon)$	$\text{Dimensions}$	$M_{max} (M_{\odot})$	$R (km)$	$R_{Sch} (km)$	$\bar{\rho} (10^{15} g cm^{-3})$	$\sigma$
1.70	5	$9.22 \times 10^6$	17.52	107.41	0.39	6.13
	6	$32.06 \times 10^{12}$	20.58	341.98	0.33	16.62
1.40	5	$6.26 \times 10^6$	14.43	88.50	0.58	6.13
	6	$17.91 \times 10^{12}$	16.95	281.65	0.48	15.72
1.10	5	$3.86 \times 10^6$	11.33	69.09	0.94	6.10
	6	$8.68 \times 10^{12}$	13.32	221.23	0.78	16.73
0.80	5	$2.04 \times 10^6$	8.24	50.52	1.78	6.13
	6	$3.34 \times 10^{12}$	9.68	160.91	1.48	16.62

## V. HIGHER DIMENSIONS

In this section, we extend our solutions to 5 and 6 dimensions to obtain the maximum mass of neutron stars by using the mentioned equation of state of neutron star matter.

First, we consider the TOV equation for gravity's rainbow without the cosmological constant. The results of our calculations show that the maximum mass of the neutron stars in 5 and 6 dimensions is not reasonable. In other words, the obtained maximum mass of neutron star is about  $10^6 M_{\odot}$  and  $10^{12} M_{\odot}$  for 5 and 6 dimensions, respectively (see table VI for more details). In order to get a better picture of the results, we calculate the Schwarzschild radius for the obtained masses in gravity's rainbow at these dimensions. The Schwarzschild radius for this gravity in higher dimensions is

$$R_{Sch} = \left( \frac{8G_d M_{eff} \Gamma \left( \frac{d-1}{2} \right)}{c^2 (d-2) \pi^{\frac{d-3}{2}}} \right)^{\frac{1}{d-3}}. \quad (50)$$

Also, we discuss the average density of the neutron star in 5 and 6 dimensions, in which can be written as

$$\bar{\rho} = \begin{cases} \frac{2M_{eff}}{\pi^2 R^4}, & d = 5 \\ \frac{15M_{eff}}{8\pi^2 R^6}, & d = 6 \end{cases}. \quad (51)$$

The results are collected in table VI. These results show that for 5 and 6 dimensions, obtained radius of the neutron stars in gravity's rainbow are within the Schwarzschild radius. In other words, these objects can not be neutron stars,

TABLE VII: Mass and radius of neutron star through the observations and theory.

<i>name</i>	<i>Observations</i>		<i>H(ε)</i>	<i>Theory</i>	
	<i>M</i> ( $M_{\odot}$ )	<i>R</i> (km)		<i>M<sub>max</sub></i> ( $M_{\odot}$ )	<i>R</i> (km)
<i>PSR J0348 + 0432</i>	2.01	13(±2)	1.195	2.01	10.06(±0.01)
<i>PSR J1614 – 2230</i>	1.97	13(±2)	1.172	1.97	9.88(±0.01)
<i>4U 1608 – 52</i>	1.74	9.3(±1)	1.035	1.74	8.71(±0.01)

TABLE VIII: Predicted radius for neutron star through the theory.

<i>name</i>	<i>Observations</i>		<i>H(ε)</i>	<i>Theory</i>	
	<i>M</i> ( $M_{\odot}$ )	<i>R</i> (km)		<i>M</i> ( $M_{\odot}$ )	<i>R</i> (km)
<i>J1748 – 2021B</i>	2.70	<i>unknown</i>	1.605	2.70	13.52(±0.01)
<i>4U 1700 – 377</i>	2.40	<i>unknown</i>	1.425	2.40	12.00(±0.01)
<i>PSR J1903 + 327</i>	1.67	<i>unknown</i>	0.994	1.67	8.37(±0.01)
<i>Cen X – 3</i>	1.49	<i>unknown</i>	0.888	1.49	7.48(±0.01)
<i>PSR B1913 + 16</i>	1.44	<i>unknown</i>	0.856	1.44	7.20(±0.01)
<i>PSR J0737 – 3039</i>	1.35	<i>unknown</i>	0.802	1.35	6.75(±0.01)
<i>PSR J0737 – 3039B</i>	1.24	<i>unknown</i>	0.738	1.24	6.21(±0.01)
<i>SMC X – 1</i>	1.04	<i>unknown</i>	0.738	1.24	6.21(±0.01)

but black holes. So the mentioned equation of state of neutron star matter could not anticipate the existence of neutron stars in higher dimensions. Therefore, the modern equations of state of neutron star matter derived from the microscopic calculations is a valid equation of state in 4 dimensions for neutron stars, but for higher dimensions, this equation of state should be modified. On the other hand, B. C. Paul et al showed that for anisotropic stars, by considering higher dimensions, the maximum mass of compact objects increases when dimensions of spacetime increase in which however attains a maximum in 5–dimensions, thereafter it decreases as one increases number of extra dimensions (see [75], for more details). As it was pointed out before, these different results may be due to different equation of states or various kinds of compact objects (anisotropic and isotropic). So it will be interesting to study the equation of state of neutron star matter in higher dimensions.

## VI. THEORY AND OBSERVATIONS

Here, we compare our results of gravity’s rainbow with observational data. For this purpose, we present these results in table VII.

As one can see, the results extracted in this theory match with the results obtained through the observations, with the exception that the radius obtained through gravity’s rainbow are smaller than the radius of compact objects. This is due to the fact that we have considered static neutron stars in our studies, so the radius of neutron stars is smaller than the radius of observational compact objects.

Evidently for static cases of observational objects, our theory under consideration predicts a set of radii which for cases such as J1748-2021B and PSR J0348+0432, predicted radii are more than 13.52 (km) and 10.06 (km), respectively (see table VIII and Fig. 15 for more details).

## VII. CONCLUSIONS

In this paper, by taking into account three models of gravity’s rainbow with (3+1)-dimensional spherically symmetric space-time, we studied the hydrostatic equilibrium equation (HEE). Then, we generalized our solutions to arbitrary  $d$ -dimensions ( $d \geq 5$ ). Next, by employing obtained HEE, we conducted a study regarding the structure of neutron stars. Regarding microscopic calculations, we used the modern equations of state of neutron star to obtain the properties

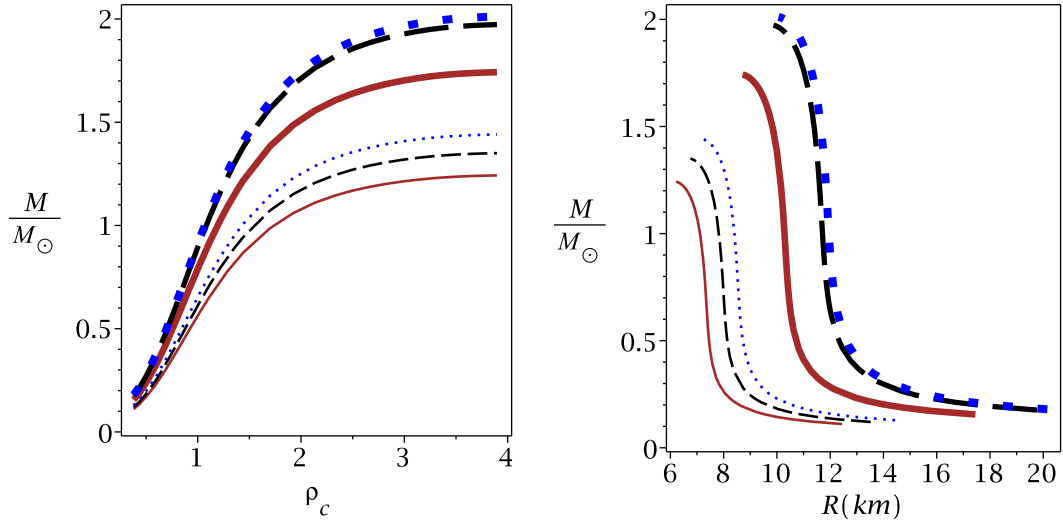


FIG. 15: Gravitational mass versus central mass density (left diagram) and radius (right diagram), for *PSR J0737 – 3039B* (continuous line), *PSR J0737 – 3039* (dashed line), *PSR B1913 + 16* (dotted line), *4U 1608 – 52* (bold-continuous line), *PSR J1614 – 2230* (bold-dashed line) and *PSR J0348 – 0432* (bold-dotted line).

of neutron star. We have studied the effects of rainbow functions on the diagrams related to the mass-central mass density relation and also the mass-radius relation of neutron star. We showed that, for  $H(\varepsilon) > 1$ , by increasing  $H(\varepsilon)$ , the maximum mass of neutron star increases to more than  $2M_\odot$  and for  $H(\varepsilon) < 1$ , the maximum mass is a decreasing function of the  $H(\varepsilon)$ .

Then, we added the cosmological constant into this gravity and examined the diagrams related to  $M-\rho_c$  and  $M-R$ . It was pointed out that for  $\Lambda < 10^{-14} m^{-2}$ , the cosmological constant ( $\Lambda$ ) does not affect the maximum mass and radius of neutron stars. On the other hand, for  $\Lambda > 10^{-14} m^{-2}$ , the mass versus radius of a neutron star may present a quark-like star behavior (see Fig. 10, for more details).

Next, we examined the effects of rainbow functions and the cosmological constant on the other properties of neutron star such as; the Schwarzschild radius, average density and strength of gravity. Also, we found a bound for rainbow function, so that for the value less than 0.6 ( $H(\varepsilon) < 0.6$ ), obtained average density for neutron stars was larger than of the central density, therefore, valid values for rainbow function in case of neutron stars are those that are larger than 0.6 ( $H(\varepsilon) > 0.6$ ). Another interesting result of this paper is the effects of the cosmological constant on the so-called gravity strength. We found that for positive of the cosmological constant, the strength of gravity is a decreasing function of the cosmological constant which leads to decreasing the maximum mass of neutron stars.

Also, we have obtained the gravitational redshift and showed that it was modified in this gravity. The gravitational redshift of neutron stars in this gravity was in the range  $z \leq 0.566$  which is lower than the upper bound on the surface redshift for a subluminal equation of state, i.e  $z = 0.851$  [116]. In addition, we have investigated the dynamical stability and found that these stars were stable against the radial adiabatic infinitesimal perturbations. Our results showed that, the neutron stars with a mass more than  $2M_\odot$ , can exist in gravity's rainbow. Then, we compared our results with the empirical data and saw that our results were compatible with those of observations. Finally, using gravity's rainbow, we obtained the radius of observational compact objects. We concluded that it is possible to obtain the mass and radius of compact objects by this theory through fine tuning.

One of the most important results of this paper is regarding determining and finding an upper and a lower limitations for rainbow function. In other words, gravity's rainbow was valid in the range of  $0.6 < H(\varepsilon) \leq 1.67$ . Furthermore, this gravity predicts a maximum limitation for the mass of neutron star which was about  $2.81M_\odot$ . Indeed, gravity's rainbow covers all neutron stars and pulsars which have the maximum mass less than  $2.81M_\odot$ .

Using gravity's rainbow we obtained relation between the maximum mass of neutron stars and the Planck mass. We showed that, this relation is modified and it depends on rainbow function. It is notable that, in the absence of rainbow function ( $H(\varepsilon) = 1$ ), this relation reduces to Einstein gravity.

Considering the gravity's rainbow and the obtained modified TOV in this gravity, we can investigate other compact objects such as white dwarf and quark stars. In addition, it is worth studying the effects of other equation of states on structure of the compact objects. On the other hand, anisotropic compact objects [117–124] in the context of gravity's rainbow are also interesting subjects. We leave these issues for future works.

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